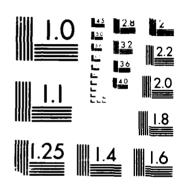
FINAL REPORT ON NEW YORK UNIVERSITY CONTRACT
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February 1, 1988

Dr. C.W. Roberson Code N00014 Office of Naval Research 800 North Quincy Street, Code 1112AI Arlington, VA 22217-5000

Re: N00014-84-K-0079, Final Report Requisition No. 4129009

Dear Dr. Roberson:

Attached please find a final report on the above-referenced contract, under the direction of Professor Harold Weitzner.

If any additional information is required please contact me at 212/998-3244.

Sincerely,

Deborah Kramer

Sponsored Research Administration

cc: Vincent Morano (1 copy)
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INTRODUCTION

The broad area of study under this grant has been the examination of fluid models of relativistic plasmas and a comparison with the kinetic relativistic Vlasov equation model. Such research is aimed at exploring the role and the limitations of relativistic fluid models in describing a relativistic, essentially collisionless, plasma. Although fluid models have the great advantages of analytic and computational simplicity compared with kinetic models, they have been used relatively sparingly in present day research. It is hoped that after careful comparison of kinetic and fluid treatments that other researchers will be encouraged to enlarge their use of fluid models in the appropriate circumstances.

Analysis of fluid plasma models serves other functions. there has been an extensive theoretical development of non-relativistic fluid dynamics and magnetohydrodynamics, many techniques are available which can be applied to relativistic plasma fluids. One can easily problems. stability. equilibrium and steady flow modes and spectral representations, of normal bifurcation of equilibria, and so on. The quantitative and qualitative understanding of such diverse phenomena greatly facilitates the sion For interpretation of numerical computations of kinetic plasmas which might $\frac{\pi}{14B}$ exhibit these properties. Even though the fluid model has significant limitations and many analytic approximations and expansions necessary for results, these failings are more than compensated by the In the ease of obtaining answers which describe complex phenomena. detailed description in the next section of work completed of cold,



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helically symmetric flows many results are exhibited. For comparison purposes helically symmetric steady flows of a warm relativistic fluid have to be constructed in conjunction with an Italian visitor, Prof. A.M. Anile of the University of Catania, Catania, Italy. This latter work has been completed but not yet published.

Many comparisons of relativistic cold fluid flows and collisionless Vlasov steady and time dependent flows have been examined. In the analysis done under this grant in the study of cool, i.e., low temperature, relativistic plasma flows, essetially no situations were found in which both the thermal effects modified the basic cold plasma flow and the lowest order fluid theory was inadequate to represent thermal effects. Thus, in steady beam flows, we have found that the usual Bennett pinch profile is not the only cool steady beam flow possible; instead, any cold plasma flow can be perturbed into a cool beam profile. Even in more complex steady flows examination in a survey paper on relativistic plasmas no major differences between cool and cold flows were found. Further, in the study of wave propagation, whenever thermal effects were critical and more than a higher order correction, higher order fluid systems were necessary. The origin of this interesting situation is apparent in the simple collisionless fluid models which have been proposed in which the thermal effects are a small correction in the momentum equation and the equations for the second moments, the equation of state, are linear in thermal corrections. Thus, the dynamics is only weakly affected by thermal effects, and the structure of thermal effects is then determined once the basic cold flow is known.

The connection between cold and cool flows was of sufficient significance that it was examined again in a less trivial circumstance than a simple cylindrical particle beam dependent on the radial coordinate only. The test case chosen is a long, thin, cylindrically symmetric electron beam that depends on both r and z. This case has been chosen as it allows more or less explicit solutions of the relativistic Vlasov equation which can be compared with the cool, fluid models. Moreover, in a thin beam it is quite possible that thermal effects might modify the basic cold fluid flow. Here as well, as indicated above no major modification were found.

II. SURVEY OF WORK

Three larger papers, "Relativistically covariant warm charged fluid modelling" (1) - I, "Plasma wave equations of state" (2) - II and, "Cold relativistic helically symmetric flows" (3) - III have been published. In addition, a survey paper, "Relativistic Plasmas" - IV will appear in the Springer-Verlag Lecture Note Series on Relativistic Fluid Dynamics. Several other related works have been published and work on stability of cold flows is in progress.

The first paper -- I -- started from the ideas of William Newcomb, ⁽⁴⁾ and developed in a different manner a cool, relativistic fluid plasma model based on a truncation scheme for equations which are the hierarchy of moments of the relativistic Vlasov equation. The role of the constraints on the moments of a relativistic distribution function is handled more thoroughly and consistently than Newcomb, and

finally a cool, relativistic fluid model is proposed. The techniques used clearly lend themselves to higher order approximations, as necessary. The model is then applied to a relativistic axisymmetric particle beam flow. In this steady flow, in cylindrical coordinates we assume that all quantities depend on ronly. We obtain the general description of such flows, and it is clear that any flows close to cold fluid flows can be extended to cool fluid flows and that no profile constraints appear such as occur in a Bennett pinch. The construction of a closed, self-consistent model, the ability to extend it to higher orders, and the relation to a cold fluid model, were the major elements of this paper.

The second paper -- II -- in large part is devoted to an analysis of linearized wave propagation based on the model of I. Away from cyclotron resonances, the odel recaptures the full panoply of plasma waves. Near cyclotron resonances the hypotheses for truncation of the linearized system fail. But if one enlarges the moment system to add third moments, then one calculates correctly cyclotron resonant waves correctly through second harmonics. In an ion and electron plasma one must examine waves at ion frequencies carefully as these waves have phase velocity much less than the electron thermal speed and the electrons cannot be considered cold. The question is examined with some care and other approaches must be taken. This paper shows very clearly the strengths and weaknesses of fluid modelling. Most waves are represented very well with a fluid model, second and higher harmonic cyclotron harmonic waves require higher order approximations, and waves at ion frequencies in an ion-electron plasma

may require dramatically different models. When the fluid models do fail, it can usually be decided within the model that the results are incorrect and other approaches are possible.

The third paper, "Cold relativistic helically symmetric steady flows"(4) III. applies methods of fluid dynamics and magnetohydrodynamics to steady, cold, relativistic, helically symmetric Such flows are of interest in free-electron lasers, an in FEL parlance we examine thick beams with self-fields included. As opposed to other treatments which depend on a paraxial approximation, we allow non-trivial radial profiles of the physical quantities. Since the flow is cold, the motion is constructed solely of single particle motions. Nonetheless, the formulation as a cold fluid greatly simplified the analysis. The flow is reduced to the solution of three second order partial differential equations, one for an electrostatic potential, one for a magnetic flux function, and one for a flow stream function. The first two equations are always elliptic, the stream function satisfied a hyperbolic equation -- akin to supersonic fluid flow. Standard perturbation methods, but ones which retain self-fields effects and radial dependent profiles, exhibit the usual cyclotron resonance enhancements to flow amplitudes, and many results well-known in the literature are easily recovered. The interesting case of a beam everywhere in cyclotron resonance is also examined. It is shown that there are bifurcated beam flows possible in which no helial magnetic or electric field is applied, yet the beam distortss into a helix. without such a self-excited helical system, a flow which is everywhere in resonance requires a much smaller applied helical system, a flow

which is everywhere in resonance requires a much smaller applied helical wiggler magnetic field in order to generate a given amplitude of helical particle flow.

The fourth paper, "Relativistic Plasmas" contains a survey of several topics. The material not contained in other works includes the construction of a relativistic adiabatic invariant for the motion of a single particle in a long, thin, steady state, relativistic beam. This adiabatic invariant is then used to construct a solution of the relativistic Vlasov equation in a non-trivial, long, thin geometry. In the realm of cold, relativistic flows a new self-excited free electron laser steady flow is constructed. In addition, cool steady flows which are cylindrically symmetric, but which depend on both r and z are examined. It is shown that these flows are small perturbations of cold steady flows. Thus, the cool relativistic plasma model proposed in I and II, does not appear to induce any fund-amental changes from a cold plasma model.

In conjunction with a visit from Prof. A.M. Anile, University of Catania, Italy, a study was started of helically symmetric flows for a warm relativistic plasma in local thermodynamic equilibrium. Most of the work has been completed although publication will be delayed. Of interest, in addition to all the topics mentioned in the cold, helically symmetric flow problem is the appearance of sonic transitions. Work is continuing in this area.

ONR Publications

- Amendt, P. and Weitzner, H., "Relativistically covariant warm charged fluid beam modeling," Phys. Fluids <u>28</u> (3), March 1985.
- 2. Amendt, P., "Plasma wave equations of state," Phys. Fluids 29 (5), May 1986.
- 3. Weitzner, H., Fruchtman, A. and Amendt, P., "Cold relativistic helically symmetric steady flows," Phys. Fluids 30 (2), February 1987.
- 4. Weitzner H., Lecture notes in "Relativistic plasmas," editors A.M. Anile and Y. Choquet-Bruhat, to be published by Springer-Verlag.
- 5. Amendt, P. and Weitzner, H., "Cool electron beam steady flows," Phys. Fluids 30 (6), June 1987.

Other Work Carried Out Under Partial ONR Support

- Amendt, P., Rahman, H.U. and Strauss, M., "Magnetic instabilities in accelerating surfaces," Phys. Rev. Lett. 53 (13), 1226 (1984).
- 2. Strauss, M., Amendt, P., Rahman, H.U. and Rostoker, N., "Line shifts in electron channeling radiation from lattice vibrations," Phys. Rev. Lett. 55 (4), 406 (1985).
- 3. Rahman, H.U., Amendt, P. and Rostoker, N., "Z-pinches with multi-ion species: Ion separation and stability," Phys. Fluids 28 (5), May 1985.
- 4. Amendt, P., Strauss, M., Rahman, H.U. and Rostoker, N., "Valence-band plasmon effects on line shifts and widths in positron planar-channeling radiation," Phys. Rev. A 33 (2), 839 (1986).

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